

**Name of the Course** : CBCS B.Sc. (Hons.) Mathematics  
**Unique Paper Code** : 32351101\_OC  
**Name of the Paper** : C1-Calculus  
**Semester** : I  
**Duration** : 3 Hours  
**Maximum Marks** : 75

Attempt any four questions out of the following. All questions carry equal marks.

1. Find the  $n^{\text{th}}$  derivative of  $y = \frac{2x}{x^2+a^2}$ . Also prove that  $y_n = \frac{(-1)^n n!}{r^{n+1}} 2\cos(n+1)\theta$ .

If  $y = \cos(m \sin^{-1}x)$  then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$

2. Sketch the graph of  $f(x) = x^4 - 4x^3 + 10$  by finding intervals of increase and decrease, critical points, relative extrema, inflection points and concavity for the given function.

Find the horizontal and vertical asymptotes to the graph of the function  $f(x) = \frac{2x}{x^2-1}$ .

Sketch the graph of the curve in polar coordinates of the curve  $r = 1 + 2\cos\theta$ .

3. Evaluate the following integrals

(i)  $\int_0^{2\pi} \sin m\theta \cos n\theta \, d\theta$ , (ii)  $\int_0^{\pi} \sin^2 6\theta \cos^4 3\theta \, d\theta$ .

Give reduction formula for  $\int \operatorname{cosec}^n \theta \, d\theta$  and find the value of  $\int \operatorname{cosec}^6 \theta \, d\theta$ .

4. Use the method of cylindrical shells to find the volume of the solid generated when the region bounded by the hyperbola  $y = \frac{1}{x}$  and the line  $y = \frac{5}{2} - x$  is revolved about the y-axis.

Find the length of the curve  $y = (x/2)^{\frac{2}{3}}$  from  $x = 0$  to  $x = 2$ .

The arc of the parabola  $y = x^2$  from (1,1) to (2,4) is rotated about y-axis. Find the area of the resulting surface.

5. Sketch the curve  $(x-3)^2 = 6(y-2)$ .

Sketch the curve  $\frac{x^2}{15} - \frac{y^2}{3} = 1$ , and label the vertices, foci and asymptotes.

Identify and sketch the curve  $x^2 - xy + y^2 - 2 = 0$ .

6. Find the limit:  $\lim_{t \rightarrow 0} \frac{\sin 2t \hat{i} + t \hat{j}}{t^2 + t - 1}$ .

A shell is fired from the ground level with muzzle speed of 650 ft/s at an angle of  $45^\circ$ . An enemy gun 21,000 ft. away fires a shot 2 seconds later and the shells collided 45 ft. above the ground at the same speed. What are the muzzle speed ( $V_0$ ) and angle of elevation ( $\alpha$ ) of the 2<sup>nd</sup> gun?

The acceleration vector of a moving particle is  $A(t) = 18t^3 \hat{i} + 3 \hat{j}$ . Find the particles position vector as a function of  $t$  if  $R(0) = \hat{i} + \hat{j}$  and  $V(0) = 1$ .